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HEAT TRANSFER FROM A HORIZONTAL CIRCULAR WIRE AT SMALL REYNOLDS AND GRASHOF NUMBERS—II

MIXED CONVECTION

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Abstract—Heat transfer from a horizontal fine cylinder by mixed, forced and free convection is investigated. Theoretical correlations of the heat transfer when forced or free convection at small Reynolds or Grashof numbers is affected by the other comparatively slight free or forced convection respectively, are given by the expansion method similar to the cases of pure convection described in Part I. The effects of the slight convection on the heat transfer are expressed systematically by a parameter of $PrRe^3/NuGr$. Experimentally, the heat-transfer behavior under corresponding mixed convections was observed by moving a wire in air enclosed in a large box either vertically downward, vertically upward or horizontally. The agreement between analyses and experiments is satisfactory especially in parallel flow.

NOMENCLATURE

a, radius of the circular wire;
D, coefficient of the expansion term in the near-field solution;
g, acceleration due to gravity;
Gr, Grashof number ($= g\beta\Delta Ta^3/\nu^2$);
h, average heat-transfer coefficient;
k, thermal conductivity;
l, length of the circular wire;
Nu, Nusselt number ($= ha/k$);
$\Delta(1/Nu_m)_G$, difference defined by equation (4.31);
$\Delta(1/Nu_m)_R$, difference defined by equation (4.32);
p, dimensionless excess pressure ($= P/\rho\bar{U}_0^2$ or $Pa^2/\rho\nu^2$);
Pr, Prandtl number;
r, θ, dimensionless cylindrical coordinates;
Re, Reynolds number ($= \bar{U}_0 a/\nu$, $U_1 a/\nu$ or $V_1 a/\nu$);
T, temperature;
t, dimensionless temperature ($= T - T_\infty/T_w - T_\infty$);
ΔT, temperature difference ($= T_w - T_\infty$);
$\bar{U}_0$, basic uniform velocity of forced convective flow;
U_1, V_1, slight uniform velocities;
U, V, velocity components;
u, v, dimensionless velocity components in x- and y-direction respectively ($= U/\bar{U}_0$, $V/\bar{U}_0$ or Ua/ν, Va/ν);
$\bar{V}$, voltage drop between both ends of the wire maintained at a constant temperature;
$\Delta\bar{V}$, voltage difference from the pure free convection case ($= \bar{V} - \bar{V}_G$);
x, y, horizontal and vertical dimensionless distances from the center of the wire respectively ($= X/a$, Y/a).

Greek symbols

β, expansion coefficient;
η, similarity variable for predominant free convection [$= (Gr\Theta_0)^{1/4} x y^{-2/5}$];
ν, kinematic viscosity;
ξ, similarity variable for predominant forced convection ($= Re^{1/2} x^{-1/2} y$);
ρ, density;
Φ, stream function for predominant forced convection ($\partial\Phi/\partial y = u, \partial\Phi/\partial x = -v$);
Ψ, stream function for predominant free convection ($\partial\Psi/\partial y = -u, \partial\Psi/\partial x = v$).

Subscripts

0, 1, 2, order of expanded terms;
c, for cross flow;
G, for pure free convection;
j, at the joining point;
m, at the arithmetic mean temperature [$= \frac{1}{2}(T_w + T_\infty)$];
p, for parallel flow;

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R, for pure forced convection;
w, at the surface of the wire;
∞, at infinity.

Superscripts

$\bar{}$, circumferential average in the near field;
$\hat{}$, circumferential average in the far field.

1. INTRODUCTION

THE PROBLEM of heat transfer from a body by mixed, forced and free convection has been studied for various geometrical configurations. The phenomena of the convective heat transfer from a horizontal or vertical wall with superiority of either one of forced or free convection to the other were analyzed by use of series expansion in the similarity solutions [1–4]. In those analyses, the dependent variables are expanded so that the zeroth-order approximation and the first-order should correspond to the predominant convection and the inferior convection, respectively. Especially in case of predominantly forced convection along a horizontal wall, the slight vertical buoyancy has an appreciable effect on the basic flow through the pressure field. Joshi and Sukhatme [14] obtained theoretically the local heat-transfer rate from a horizontal cylinder to a transverse flow by mixed convection, assuming a thin curved boundary-layer around the cylinder at sufficiently large Reynolds and Grashof numbers and using the method similar to the cases of mixed convection along a plane wall.

In regard to heat transfer from a very small body to a surrounding fluid, the parallelly mixed convection from a sphere at small Reynolds and Grashof numbers was solved by Hieber and Gebhart [5] only for the range of Reynolds and Grashof numbers limited by the relation $Gr = 0(Re^2)$ as $Re \to 0$, by use of the method of matching the solutions in the inner and outer regions. The incipient effect of buoyancy on the same-directional basic flow past a horizontal cylinder was investigated by Wood [16] mainly with series expansion in the similarity solutions, which is the same as the far-field solution for the case of forced convection with slight parallel free convection in the present study. Further, he discussed the incipient effect on the slanted upward basic flow at an acute angle α to the vertical line by putting $Gr \cdot \cos\alpha$ in place of Gr and estimating the change in pressure across the entire width of the slanted upward wake.

There are a few experimental reports about the heat transfer from a cylinder by mixed convection when the Reynolds and Grashof numbers are relatively large [6–8]. Those experimental data are scattered considerably when correlated with the parameter Gr/Re^2 and the angle between the directions of forced and free convections. In the experiments made by Hatton, James and Swire [9] for the range of medium Reynolds and Grashof numbers ($10^{-3} < Gr_\infty < 10$), it was attempted to express all the Nusselt numbers for the parallel, contra and cross combined-convections as a vectorial sum of the forced and free convective heat-transfer correlations. From the physical point of view, however, the concept of the vectorial sum seems to be irrelevant except for the case of parallel flow.

At relatively small Reynolds and Grashof numbers ($0{\cdot}5 \times 10^{-3} \leqslant 8PrGr_\infty \leqslant 6 \times 10^{-3}$ for $Pr = 6{\cdot}3$; $0{\cdot}5 \times 10^{-4} \leqslant 8PrGr_\infty \leqslant 6 \times 10^{-4}$ for $Pr = 63$), Gebhart and Pera [10] carried out parallelly mixed convection experiments, but they did not propose any suitable parameters to correlate the results. The buoyancy effect on the forced convective heat transfer in cross flow at small Reynolds and Grashof numbers was investigated experimentally by Collis and Williams [11], who derived a rough criterion for the onset of buoyancy effect that $Re_m = 1{\cdot}85\, Gr_\infty^{0{\cdot}39}$.

It is the purpose of the present study to obtain the systematical synopsis of the heat-transfer characteristics from a horizontal infinite fine wire by mixed convections at small Reynolds and Grashof numbers in representative combinations of the directions of the forced flow and the buoyant force, that is, in parallel, contrary and cross flows. When either one of the forced or free convection is subordinate to the other, theoretical solutions can be examined in the similar manner to the cases of pure convection described in Part I. Experiments were carried out for the three types of mixed convections by use of a horizontal long fine wire moved in air enclosed in a large box. The differences of the heat-transfer rate from that by pure free convection at the same Grashof number were precisely measured so as to examine the validity of the analytical results in detail.

2. DESCRIPTION OF ANALYSES

Taking the same configuration of the problem as the cases of pure convection described in Part I, we can use the same governing equations and then the variables should be rendered non-dimensional by the units corresponding to whether the forced or free convection is predominant. For cases of the predominant forced convection, the basic uniform flow is taken in the x-direction and its stream function is defined by $\partial\Phi/\partial y = u$ and $\partial\Phi/\partial x = -v$, while for cases of the predominant free convection, the upward convective flow is taken in the y-direction with the stream function defined by $\partial\Psi/\partial y = -u$ and $\partial\Psi/\partial x = v$. The boundary conditions are considered as

$$\left.\begin{array}{lll} t = 1, u = v = 0 & \text{on} & r^2 = x^2 + y^2 = 1, \\ t \to 0, u \to 1 \text{ or } u \text{ or } v \to Re & \text{as} & r \to \infty, \end{array}\right\}$$

together with the other suitable behavior of the velocity field at infinity. As in Part I, the surrounding temperature-field about the heated cylinder is divided into the two fields, the near conduction-field and the far convection-field, which can be joined up smoothly to determine the heat-transfer characteristics.

Since the near-field solution is irrelative to the convective flow direction, it can be identical with that for pure convection so that the circumferential average temperature defined by

$$\bar{t} = \frac{1}{2\pi}\int_0^{2\pi} t\,\mathrm{d}\theta$$

is approximately given by either

$$\bar{t} = 1 - Nu\ln r + D\int_1^r \frac{1}{r}(r-1)^3\,\mathrm{d}r, \qquad (2.1)$$

or

$$\bar{t} = 1 - Nu\ln r, \qquad (2.2)$$

(in detail, see Part I). Comparison with experimental results indicated in Part I that equation (2.1) is preferably used when the free convection is dominant, while equation (2.2) is favorable for cases of the predominant forced convection.

The far-field solutions are obtainable by the consideration almost similar to that of the mixed convection along a plane wall. The dependent variables in the far field are to be expanded so that the zeroth-order approximation should represent the predominant convection and the first-order the other inferior convection. The far-field solution including the approximations up to the second-order could be joined up smoothly to the near-field solution to give the effect of the slightly superposed convection on the heat-transfer correlation.

3. DESCRIPTION OF EXPERIMENTS

(i) *Equipment*

The equipment consisted of a large box maintaining a still air in it, a long fine circular wire mounted on a support-rig which is movable at a constant speed, an electrical heat-supply and a measurement system.

The test enclosure of the wooden box was 180 cm in length and 80 × 80 cm^2 in cross-section. The temperature of the air in the box was measured with fifteen thermo-couples placed at various positions, within accuracy of ±0·025°C.

Tungsten wires of diameters from 0·00064 to 0·00262 cm and of lengths from 1·5 to 29 cm were used as test specimens. The diameter was measured by means of microscopic photographs, of which accuracy was about 5 per cent for the finest wire due to the obscurity of the fringes of wire in photographs. The deviation of the cross-section shape from a circle was relatively small. The length of wires was measured by a travelling microscope. The calibration for the temperature coefficients of electric resistance was carefully carried out in a small metallic box immersed in an oil bath maintained at a constant temperature within ±0·05°C and showed similar characteristics to those reported in the reference [12] for fine drawn tungsten-wires. The temperature coefficients shown in Table 1 varied with a discontinuity at the diameter of 0·0010 cm because wires of the diameter less than 0·0010 cm were refined with the electrolytic etching. The surface contamination of wires was not apparent throughout the period of these tests.

Table 1. Characteristics of wires used

Diameter $2a$ (cm)	Temperature coefficient of resistance	Aspect ratio ($l/a \times 10^{-4}$)
0·00064	0·00355	0·45, 0·58, 1·7, 2·1
(0·00077	0·00366)	
0·00088	0·00377	0·50, 1·1, 2·2, 2·3, 2·5
0·00164	0·00366	0·55, 1·1, 2·1, 2·4, 3·9
(0·00222	0·00393)	
0·00262	0·00409	0·51, 0·57, 2·2

The shape of wire-supports as shown in Fig. 1 was employed to reduce the influence of disturbance caused by its prongs which were common to the three types

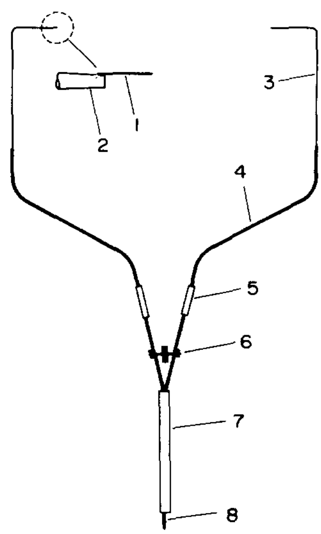


FIG. 1. Wire support geometry (the whole height = 35–45 cm) 1, Tungsten wire; 2, About 0·02 cm dia steel prong; 3, 0·1 cm dia steel rod; 4, 0·25 cm dia brass pipe; 5, Bakelite insulation; 6, Screw adjustment of tension; 7, 0·6 cm dia sustained pipe; 8, Electric jake.

of mixed convections. The test wire was mounted by spark welding on the support prongs of 0·1 cm dia steel rod and tension was applied by a screw adjustment.

The wire was moved linearly at a constant speed by a long big screw attached to the support. The time lag attaining to the constant speed was made negligible by use of a big-powered motor and an electromagnetic clutch. The movable length of the wire was about 80 cm in the longitudinal direction of the box.

A commercially available hot-wire anemometer of constant-temperature type with the chopper-type DC amplifier was used as the electric power supply to the wire. The voltage drop between both ends of the wire and the voltage drop corresponding to the current through it were measured by a digital voltmeter of five-digits display at 1·3 samplings/s and recorded with a vigicorder through a Digital-to-Analogue converter. The wire speeds were counted digitally with a phototransistor.

(ii) *Experimental procedure and observation*

At each constant temperature of the wire, namely at each fixed Grashof number, experiments were run in series from pure free convection through mixed convection to pure forced convection in about 3 h. In order to calm the air in the box after each run, a sufficient time interval of 10–20 min was taken before starting next run. The variation in the electric resistance of the wire within one test run was too small to be detected.

The temperature difference between the wire and the air in the box, $\Delta T = T_w - T_\infty$, was maintained as small as 12–30°C in order to reduce the temperature loading effect. The air in the box had a small vertical temperature difference and a negligible horizontal temperature difference. The vertical temperature difference was less than about 0·4°C even when the room temperature around the box changed considerably. The air temperature in the box increased slightly about 0·5°C during a series of test runs.

For the cases of mixed convections in parallel and contrary flows, the equipment was erected so that the longitudinal direction of the box, namely the moving direction of the wire, could be in a vertical line. In these cases, by the small temperature stratification, the voltage drop was slightly influenced as seen in Figs. 2 (a) and (b), where the upper two of four or five digits displayed on the digital voltmeter were cut off to record only the lower third and fourth digits analogically. The variation in the infinity temperature or the voltage drop with the height resulted in a small variation in the basic Grashof and Nusselt numbers, among which the latter was less influenced. The difference between two parallel inclined lines as shown in the figures was considered as the voltage difference between mixed and free convections, $\Delta \bar{V} = \bar{V} - \bar{V}_G$. Figure 2 (c) is a typical record of the voltage drop for the cross-flow convection.

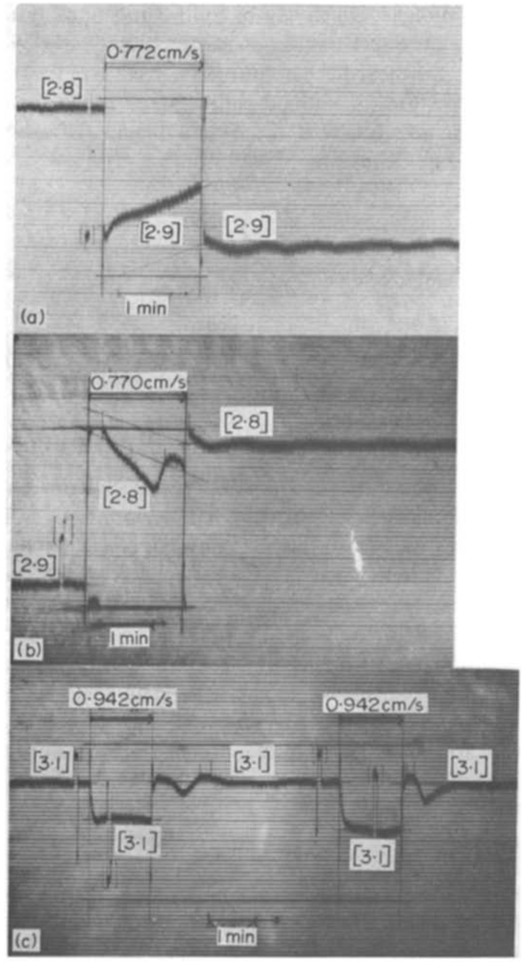


FIG. 2. Example of the trace of voltage drop. (a) Parallel flow; (b) Contrary flow; (c) Cross flow.

Mechanical vibration was sometimes observed at the support in the case of nearly pure forced convection with the longest wires of diameters 0·00164 and 0·00262 cm. In this case, similar fluctuation of the supplied voltage at frequency of about 30 Hz was also observed on the oscilloscope. However, it had no significant influence upon the output of the digital voltmeter which was the average in 200 ms.

The experiments were carried out in the range of $Gr_\infty = 10^{-7}$ to 10^{-5} and $Re_m = 0{,}4 \times 10^{-4}$ to 5×10^{-2}. The main source of errors in Gr_∞ and Re_m was the measurement of the diameter, while the stratification of the air temperature and the estimation of the longitudinally averaged wire-temperature caused relatively small errors in Gr_∞ and Re_m. The physical properties of the air used in non-dimensional parameters were evaluated in the same manner as the cases of pure convection, that is, at T_∞ or T_m. The subscripts ∞ and m denote the properties evaluated at the infinity temperature T_∞, which is the temperature at the distance of 10 cm from the side wall of the box in the horizontal plane including the wire, and at the arithmetic mean of the wire and infinity temperatures, $\frac{1}{2}(T_w + T_\infty)$, respectively.

For pure convections, the effect of the metallic thermoconduction from the wire to the support prongs had been analytically corrected [13]. Such correction had derived nearly constant Nusselt numbers at a given Grashof number or Reynolds number for $l/a = 5000$ to 40 000, being less than 0·5 per cent at $l/a = 20\,000$. The Nusselt numbers for mixed convections were also corrected in the same way.

The effect of the temperature jump at the surface due to the fineness of wires was considered to be negligible because the corresponding Knudsen number was about 0·02 for the finest wire. Even at the maximum Knudsen number, the correction was less than 1 per cent after the evaluation by Collis and Williams [11].

4. PARALLEL FLOW

(i) Free convection with slight parallel forced convection

We consider a comparatively slight uniform flow in the opposite direction of gravity, namely in the y-direction, in addition to a predominant free convection flow in the same direction at small Grashof numbers. The uniform flow has the velocity at infinity, $v(x = \pm\infty) = V_1 a/\nu = Re$, which cannot be satisfied with the expansion terms for the case of pure free convection described in Part I. Dependent variables in the far field are to be expanded so that the zeroth-order approximation could represent the pure free convection and the first-order approximation the uniform flow with the above boundary condition. With the same similarity variable η as the case of pure free convection, the stream function Ψ, the temperature t and the excess pressure p will be expanded with respect to $Rey^{-1/5}(\ll 1)$, as follows:

$$\Psi = y^{3/5}\{\pi_0\cdot\psi_0(\eta) + Rey^{-1/5}\pi_{1p}\cdot\psi_{1p}(\eta) + (Rey^{-1/5})^2\pi_{2p}\cdot\psi_{2p}(\eta) + \ldots\}, \quad (4.1)$$

$$t = y^{-3/5}\{\Theta_0\cdot\theta_0(\eta) + Rey^{-1/5}\Theta_{1p}\cdot\theta_{1p}(\eta) + (Rey^{-1/5})^2\Theta_{2p}\cdot\theta_{2p}(\eta) + \ldots\}, \quad (4.2)$$

$$p = y^{-4/5}\{\Gamma_{0p}\cdot\gamma_{0p}(\eta) + Rey^{-1/5}\Gamma_{1p}\cdot\gamma_{1p}(\eta) + (Rey^{-1/5})^2\Gamma_{2p}\cdot\gamma_{2p}(\eta) + \ldots\}, \quad (4.3)$$

where $\eta = (Gr\Theta_0)^{1/4}xy^{-2/5}$. The approximations up to the fifth-order produce a set of the boundary-layer type equations and the sixth-order corresponds to the first-order approximation for the case of pure free convection.

The zeroth-order approximation is the same as that for the case of pure free convection.

$$-\frac{3}{5}\psi_0\psi_0'' + \frac{1}{5}(\psi_0')^2 = \psi_0''' + \theta_0, \quad (4.4)$$

$$\frac{3}{5}\psi_0\theta_0 = -\frac{1}{Pr}\theta_0', \quad (4.5)$$

$$\left.\begin{aligned} &\eta = 0: \psi_0 = \psi_0'' = 0,\ \theta_0 = 1;\\ &\eta\to\infty: \psi_0' \to 0;\\ &\Theta_0 = \left\{\frac{1}{\pi}PrGr^{1/4}Nu^{-1}\int_0^\infty \psi_0'\theta_0\,d\eta\right\}^{-4/5}. \end{aligned}\right\} \quad (4.6)$$

The equations and the boundary conditions for the first-order approximation become

$$-\frac{2}{5}\psi_0''\psi_{1p} + \frac{1}{5}\psi_0\psi_{1p}'' = \psi_{1p}''' + \theta_{1p}, \quad (4.7)$$

$$\frac{3}{5}\psi_{1p}'\theta_0 + \frac{3}{5}\psi_{1p}\theta_0' + \frac{4}{5}\psi_0'\theta_{1p} + \frac{3}{5}\psi_0\theta_{1p}' = -\frac{1}{Pr}\theta_{1p}'', \quad (4.8)$$

$$\left.\begin{aligned} &\eta = 0: \psi_{1p} = \psi_{1p}'' = \theta_{1p}' = 0;\\ &\eta\to\infty: \psi_{1p}' \to 1,\ \theta_{1p}\to 0;\\ &\int_0^\infty (\psi_{1p}'\theta_0 + \psi_0'\theta_{1p})\,d\eta = 0, \end{aligned}\right\} \quad (4.9)$$

where $\pi_{1p} = (Gr\Theta_0)^{-1/4}$ and $\Theta_{1p} = Gr^{-1/2}\Theta_0^{1/2}$. The boundary condition $\psi_{1p}'(\infty) = 1$ means that the v-velocity component of the first-order has the magnitude or order of unity. The last integral condition is always satisfied with equation (4.8). Equations (4.7) and (4.8) can be solved numerically by means of superposition because of their linearity. The upward velocity component ψ_{1p}' and the temperature θ_{1p} calculated by the Runge–Kutta–Gill method show that the upward flow is slightly stronger and the temperature is slightly lower than those for pure free convection (the zeroth-order approximation) although $\psi_{1p}'(0) < 1$.

Consequently, the circumferential average temperature

$$\hat{t} \equiv \frac{1}{\pi r}\int_0^\infty t\,dx$$

is

$$\hat{t} = A_0\frac{1}{Gr}(NuGr)^{3/5}r^{-1}y^{-1/5} \times \left\{1 - \frac{A_{1p}}{A_0}Re(NuGr)^{-2/5}y^{-1/5}\right\},$$

$$A_0 \equiv \frac{1}{\pi}\left(\frac{1}{\pi}Pr\int_0^\infty \psi_0'\theta_0\,d\eta\right)^{-3/5}\left(\int_0^\infty \theta_0\,d\eta\right),$$

$$A_{1p} \equiv \frac{1}{\pi}\left(\frac{1}{\pi}Pr\int_0^\infty \psi_0'\theta_0\,d\eta\right)^{-1/5}\left(\int_0^\infty \theta_{1p}\,d\eta\right). \quad (4.10)$$

The necessary and sufficient conditions that equations (2.1) and (4.10) are to be joined up smoothly at $r = r_j$ are

$$\bar{t} = \hat{t}, \quad \frac{d\bar{t}}{dr} = \frac{d\hat{t}}{dr}, \quad \frac{d^2\bar{t}}{dr^2} = \frac{d^2\hat{t}}{dr^2}.$$

The unknowns r_j, D and Nu are thus determined under the conditions that $r \equiv y \gg 1$ and that the second term of equation (4.10) is sufficiently small compared with the first, namely $Re(NuGr)^{-1/3} \ll 1$.

$$r_j = C_A (NuGr)^{-1/3}\left\{1 + C_{A1p}\left(\frac{PrRe^3}{NuGr}\right)^{1/3}\right\},$$
$$C_A \equiv \left(\frac{42}{25}A_0\right)^{5/6},$$
$$C_{A1p} \equiv \frac{55}{54}\left(\frac{42}{25}A_0\right)^{-1/6} A_0^{-1}A_{1p}Pr^{-1/3}, \quad (4.11)$$

$$D = C_B\frac{1}{Gr}(NuGr)^2\left\{1 + \frac{63}{22}C_{A1p}\left(\frac{PrRe^3}{NuGr}\right)^{1/3}\right\},$$
$$C_B \equiv \frac{12}{25}A_0\left(\frac{42}{25}A_0\right)^{-7/2}, \quad (4.12)$$

$$\frac{1}{Nu} = \frac{1}{3}\ln E - \frac{1}{3}\ln(NuGr) + Q_p\left(\frac{PrRe^3}{NuGr}\right)^{1/3},$$
$$E = 3{\cdot}1 \times (Pr + 9{\cdot}4)^{1/2}Pr^{-2},$$
$$Q_p \equiv \frac{170}{189}\left(\frac{42}{25}\right)^{-1/6} A_0^{-7/6}A_{1p}Pr^{-1/3}. \quad (4.13)$$

The third term of equation (4.13), $(PrRe^3/NuGr)^{1/3}$, represents the effect of a slight uniform forced flow on the free convection, being proportional to Re. The calculated results are shown in Fig. 3(a). The relationships of joining equations (2.2) and (4.10) give the coefficient Q_p different only about 3·5 per cent from that by equation (4.13) which joins equations (2.1) and (4.10). Since the coefficient Q_p has the negative sign, the heat-transfer rate is slightly larger than that for pure free convection. The coefficient Q_p is not a noticeable function of Pr as seen in Fig. 3(a), so that the effect of Pr on Nu is considered to be expressed approximately in the form of $(PrRe^3/NuGr)^{1/3}$. The restrictions for the results are derived from the conditions that $r_j \gg 1$, $Rey_j^{-1/5} \ll 1$ and $Re(NuGr)^{-1/3} \ll 1$, and are expressed as

$$Pr^2Gr \leqslant 10^{-3}, \quad \left(\frac{PrRe^3}{NuGr}\right)^{1/3} \ll 1. \quad (4.14)$$

(ii) *Forced convection with slight parallel free convection*

Next, we consider a slight buoyancy force in the x-direction in addition to the predominant uniform flow in the same direction. Then, the buoyancy term $(Gr/Re^2)t$ must be contained in the momentum equation in the x-direction.

If the same similarity variable ξ as the pure forced convection is taken, the dependent variables in the far field may be expanded so as to make the zeroth-order terms correspond to the pure forced convection and the first-order to the buoyancy force. Therefore, the stream function Φ, the temperature t and the excess pressure p will be expanded with respect to $Grx^{1/2}(\ll 1)$ as follows.

$$\Phi = x^{1/2}\{F_0\cdot f_0(\xi) + Grx^{1/2}F_{1p}\cdot f_{1p}(\xi) + (Grx^{1/2})^2F_{2p}\cdot f_{2p}(\xi) + \ldots\}, \quad (4.15)$$

$$t = x^{-1/2}\{G_0\cdot g_0(\xi) + Grx^{1/2}G_{1p}\cdot g_{1p}(\xi) + (Grx^{1/2})^2G_{2p}\cdot g_{2p}(\xi) + \ldots\}, \quad (4.16)$$

$$p = x^{-1}\{H_{0p}\cdot h_{0p}(\xi) + Grx^{1/2}H_{1p}\cdot h_{1p}(\xi) + (Grx^{1/2})^2H_{2p}\cdot h_{2p}(\xi) + \ldots\}, \quad (4.17)$$

where $\xi = Re^{1/2}x^{-1/2}y$. These expansions are valid only for the region of $1 \ll r \ll Gr^{-2}$. It is noted that these may be equivalent to the re-expansions of the zeroth-order approximation terms for the case of pure forced convection (i.e. these expansions hold only in the boundary-layer type equation).

The solutions for the zeroth-order have the same form as the case of pure forced convection.

$$f_0 = \xi, \quad (\equiv \phi_0), \quad (4.18)$$

$$g_0 = G_{000}\exp\left(-\frac{1}{4}Pr\xi^2\right), \quad (\equiv G_{000}\chi_0), \quad (4.19)$$

$$G_{000} \equiv g_0(0) = \sqrt{\pi}\,Pr^{-1/2}Re^{-3}Nu, \quad (4.20)$$

where $F_0 = Re^{1/2}$ and $G_0 = Re^{5/2}$.

The equations and the boundary conditions for the first-order are

$$\frac{1}{2}f_0'f_{1p}' - \frac{1}{2}f_0f_{1p}'' - f_0''f_{1p} = f_{1p}''' + g_0, \quad (4.21)$$

$$-\frac{1}{2}f_{1p}'g_0 - \frac{1}{2}f_0g_{1p}' - f_{1p}g_0' = \frac{1}{Pr}g_{1p}'', \quad (4.22)$$

$$\left.\begin{aligned} &\xi = 0: f_{1p} = f_{1p}'' = g_{1p}' = 0; \\ &\xi \to \infty: f_{1p}' \to 0, \quad g_{1p} \to 0, \end{aligned}\right\} \quad (4.23)$$

where $F_{1p} = 1$ and $G_{1p} = Re^3$. By use of equations (4.18), (4.19) and (4.23), equations (4.21) and (4.22) are integrated as follows:

$$f_{1p}' = G_{000}\left[\frac{2}{1-Pr}\exp\left(-\frac{1}{4}Pr\xi^2\right) - \frac{2\sqrt{Pr}}{1-Pr}\exp\left(-\frac{1}{4}\xi^2\right) + \xi\left\{\frac{Pr}{1-Pr}\int_0^\xi \exp\left(-\frac{1}{4}Pr\xi^2\right)d\xi - \frac{\sqrt{Pr}}{1-Pr}\int_0^\xi \exp\left(-\frac{1}{4}\xi^2\right)d\xi\right\}\right],$$
$$(\equiv G_{000}\phi_{1p}), \quad Pr \neq 1, \quad (4.24)$$

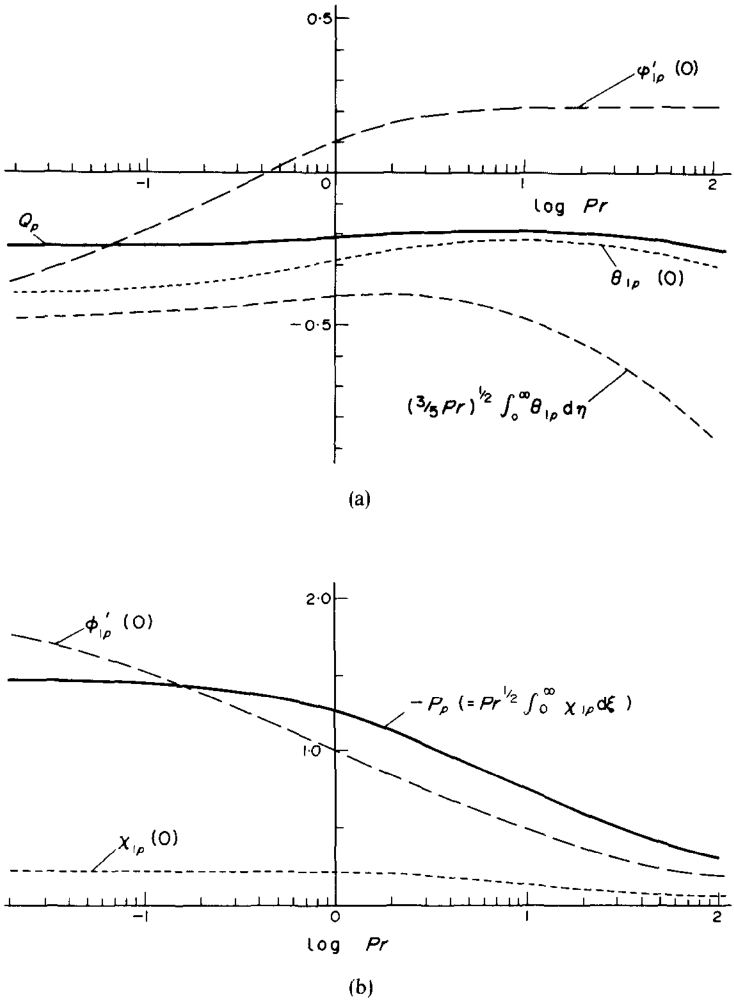


FIG. 3. Calculated results in parallel flow. (a) Free convection with slight parallel forced convection; (b) Forced convection with slight parallel free convection.

$$f'_{1p} = G_{000}\exp\left(-\frac{1}{4}\xi^2\right), \quad (\equiv G_{000}\phi_{1p}), \qquad Pr = 1, \quad (4.25)$$

$$g'_{1p} = -Pr\exp\left(-\frac{1}{4}Pr\xi^2\right) \times\left[\int_0^\xi\left\{\left(\frac{1}{2}f'_{1p}g_0 + f_{1p}g'_0\right)\exp\left(-\frac{1}{4}Pr\xi^2\right)\right\}d\xi\right] \quad [\equiv (G_{000})^2\chi_{1p}]. \quad (4.26)$$

From numerical results, it is found that the velocity component in the x-direction becomes slightly larger and the temperature becomes lower than those for the case of pure forced convection (the zeroth-order approximation).

Consequently, the circumferential average temperature is given by

$$\hat{t} \equiv \frac{1}{\pi r}\int_0^\infty t\,dy = \frac{Nu}{PrRe} \times\left\{1 + NuRe^{-5/2}(Grx^{1/2})\int_0^\infty \chi_{1p}d\xi\right\}r^{-1}. \quad (4.27)$$

The condition that equations (2.2) and (4.27) are joined up smoothly at $r = r_j$ gives

$$\bar{t} = \hat{t}, \quad \frac{d\bar{t}}{dr} = \frac{d\hat{t}}{dr},$$

from which the unknowns r_j and Nu are determined with the restriction $r \equiv x \gg 1$ and the approximation

$$NuRe^{-5/2}Grx^{1/2}\int_0^\infty \chi_{1p}\,d\xi \ll 1.$$

$$r_j = \frac{1}{PrRe}\left(1+\frac{1}{2}P_p\frac{NuGr}{PrRe^3}\right),$$

$$P_p = Pr^{1/2}\int_0^\infty \chi_{1p}\,d\xi, \quad (4.28)$$

$$\frac{1}{Nu} = 1-\ln(PrRe)+P_p\frac{NuGr}{PrRe^3}. \quad (4.29)$$

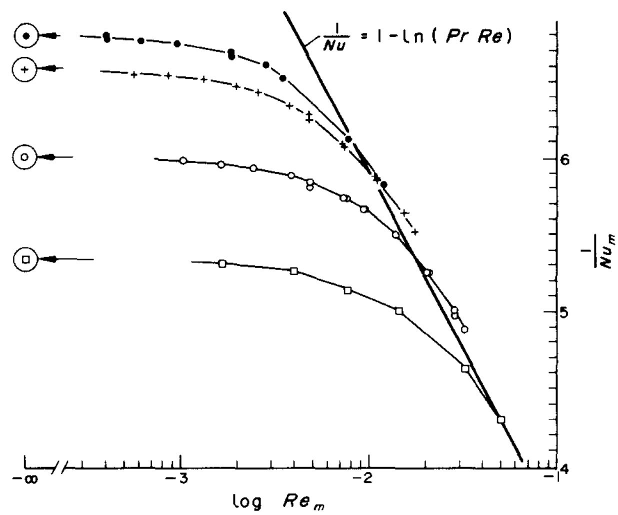


FIG. 4. Heat-transfer correlation in parallel flow for $20\,000 \leqslant l/a \leqslant 25\,000$.

●, $a = 0{\cdot}00032$ cm; $Gr_\infty = 1{\cdot}42 \times 10^{-7}$;
+, $a = 0{\cdot}00044$ cm; $Gr_\infty = 3{\cdot}2 \times 10^{-7}$;
○, $a = 0{\cdot}00082$ cm; $Gr_\infty = 1{\cdot}67 \times 10^{-6}$;
□, $a = 0{\cdot}00131$ cm; $Gr_\infty = 7{\cdot}5 \times 10^{-6}$.

The third term of equation (4.29), $NuGr/PrRe^3$, implies the effect of slight parallel free convection on the forced convection, being proportional to Gr. Nu increases with Gr since the coefficient P_p is negative as shown in Fig. 3 (b). The restriction for equation (4.29) may be simplified as follows:

$$PrRe \leqslant 1, \quad \frac{NuGr}{PrRe^3} \ll 1, \quad (4.30)$$

which satisfy incidentally the peculiar condition $Grr_j^{1/2} \ll 1$.

(iii) *Experimental results and comparison*

The horizontal test-wire was moved downward vertically and the voltage drop between both ends of the wire to maintain it at a constant temperature was measured as in Fig. 2 (a). The curves of $1/Nu_m$ against $\log(Re_m)$ at a fixed Grashof number are shown in Fig. 4 for $20\,000 \leqslant l/a \leqslant 25\,000$. These experimental results were almost independent of the aspect ratio of wire in the range of $l/a = 5000$ to $40\,000$ and they were not apparently influenced with the diameters of horizontal prongs of the support (0·02 to 0·5 cm dia). Even if a comparatively small box of test-section area $25 \times 40\,\text{cm}^2$ was used, the same results could be obtained as far as the vertical prongs of the support were placed at the distance larger than 5 cm apart from the side wall of the box. Although the variation in $1/Nu_m$ against $\log(Re_m)$ is considerably smooth, each curve at a fixed Grashof number does not always approach a common line asymptotically as for the pure forced convection. This is considered to result from measurement error of the diameter of wire.

The comparison of the experimental result with equation (4.13) for free convection affected by slight parallel forced convection is shown in Fig. 5 (a) for $20\,000 \leqslant l/a \leqslant 25\,000$, where the solid line represents the third term of equation (4.13) for $Pr = 0{\cdot}72$. The ordinate $\Delta(1/Nu_m)_G$ is defined by

$$\Delta\left(\frac{1}{Nu_m}\right)_G = \frac{1}{Nu_m}-\left(\frac{1}{Nu_m}\right)_G, \quad (4.31)$$

where the subscript G denotes the value for the case

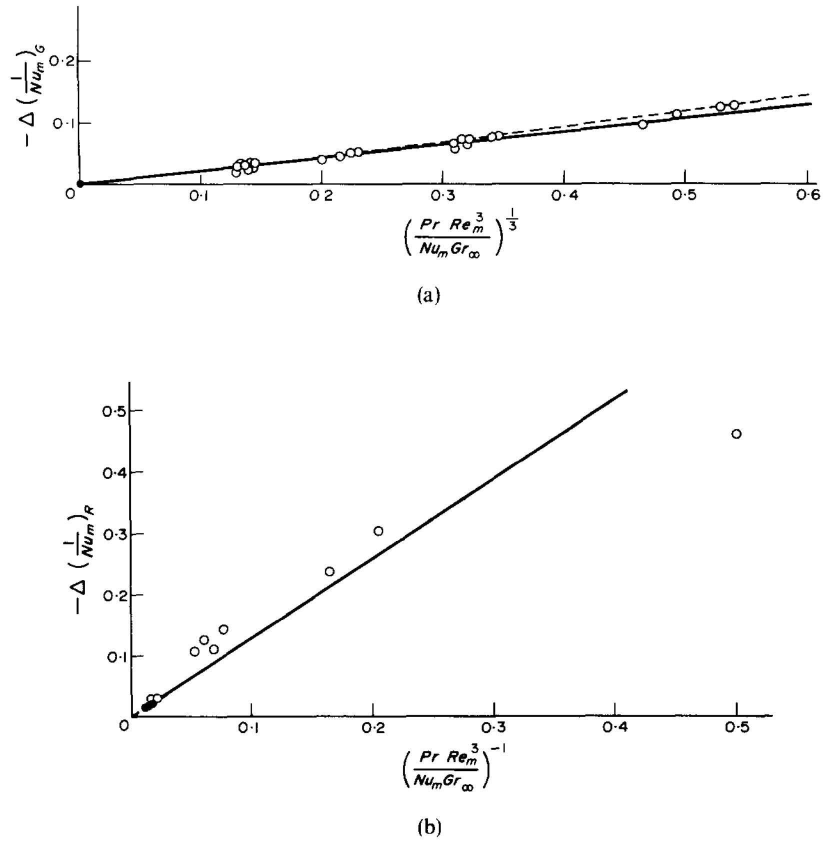


FIG. 5. Comparison of the analyses and experiments ($20000 \leqslant l/a \leqslant 25000$). (a) Free convection with slight parallel forced convection; (b) Forced convection with slight parallel free convection.

of pure free convection. The agreement between both results is considerably good in the region of $(PrRe_m^3/Nu_m\,Gr_\infty)^{1/3} \leqslant 0{\cdot}6$.

The comparison of the experimental result with equation (4.29) for forced convection affected by slight parallel free convection is shown in Fig. 5 (b) for $20000 \leqslant l/a \leqslant 25000$, where the solid line represents the third term of equation (4.29) for $Pr = 0{\cdot}72$. To eliminate the deviation due to the error of the diameter measurement, we rewrite equation (4.29) as

$$\frac{1}{Nu_m} = C - \ln(PrRe) + P_p\left(\frac{PrRe_m^3}{Nu_m\,Gr_\infty}\right)^{-1},$$

with an introduction of constant C, which is to be determined so as to make the curve fit the experimental point at the largest Reynolds number in a definite Grashof number, and then use the ordinate defined by

$$\Delta\left(\frac{1}{Nu_m}\right)_R = \frac{1}{Nu_m} - \left(\frac{1}{Nu_m}\right)_R, \qquad (4.32)$$

where

$$\left(\frac{1}{Nu_m}\right)_R = C - \ln(PrRe).$$

The solid points in Fig. 5 (b) represent the refered points to determine C in the above equation, which is close to unity. The agreement in Fig. 5 (b) is also satisfactory in the region of $(PrRe_m^3/Nu_m\,Gr_\infty)^{-1} \leqslant 0{\cdot}25$.

Here, we consider the comparably mixed free and forced convection. If we put $Nu_G = Nu_R$ with $C = 1$, the following relation will be obtained:

$$\frac{PrRe^3}{NuGr} = \frac{e^3}{3{\cdot}1(9{\cdot}4+Pr)^{0{\cdot}5}}, \quad (\equiv N_{eq}). \qquad (4.33)$$

The parameter $PrRe^3/NuGr$ implies the ratio of the heat-transfer rate of pure forced convection to that of pure free convection. Then, an attempt to correlate the difference $\Delta(1/Nu_m)_G$ with $(PrRe_m^3/Nu_m\,Gr_\infty)^{1/3}$ for the comparably mixed convection of parallel flow is made in Fig. 6. It is found that the experimental points in the figure can be expressed by the following correlation

$$-\Delta\left(\frac{1}{Nu_m}\right)_G = Q_p\left(\frac{PrRe_m^3}{Nu_m Gr_\infty}\right)^{1/3} - \left\{Q_p\left(\frac{PrRe_m^3}{Nu_m Gr_\infty}\right)^{1/3}\right\}^2. \quad (4.34)$$

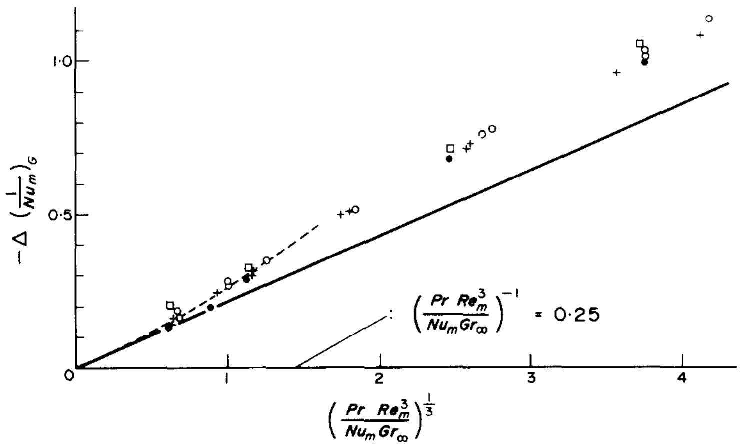


FIG. 6. Comparably mixed convection in parallel flow. ●, +, ○, □, see the legend of Fig. 4.

This correlation, which is shown by the dotted line in Fig. 6, gives a good approximation in the region of $(PrRe_m^3/Nu_m Gr_\infty)^{1/3} \leqslant 1{\cdot}6$, although this does not continuously connect with that for the case of forced convection with slight parallel free convection.

The comparison of the correlation (4.34) with the experimental points by Gebhart and Pera [10] is shown in Fig. 7 for $Pr = 6{\cdot}3$ and 63, where the solid straight line is the exter-polated line of equation (4.13) and the dotted line is equation (4.34). Although they did not directly measure the difference from the pure free convection and had relatively large values of Pr^2Gr_∞, the agreement especially at $Pr = 6{\cdot}3$ is good.

5. CONTRARY FLOW

(i) Free convection with slight contrary forced convection

Here, we consider a comparatively slight uniform flow in the opposite y-direction, namely in the gravitational direction, in addition to the predominant free convection in the y-direction. The corresponding boundary condition becomes $v(x \to \pm\infty) = -V_1 a/\nu = -Re$. Therefore, except with the negative sign of the Reynolds number, the same result (4.13) can be obtained.

$$\frac{1}{Nu} = \frac{1}{3}\ln E - \frac{1}{3}\ln(NuGr) - Q_p\left(\frac{PrRe^3}{NuGr}\right)^{1/3}, \quad (5.1)$$

in which the restrictions are

$$Pr^2Gr \leqslant 10^{-3}, \quad \left(\frac{PrRe^3}{NuGr}\right)^{1/3} \ll 1. \quad (4.14)$$

(ii) Forced convection with slight contrary free convection

For the same consideration as the above for predominant free convection, the following results are obtained:

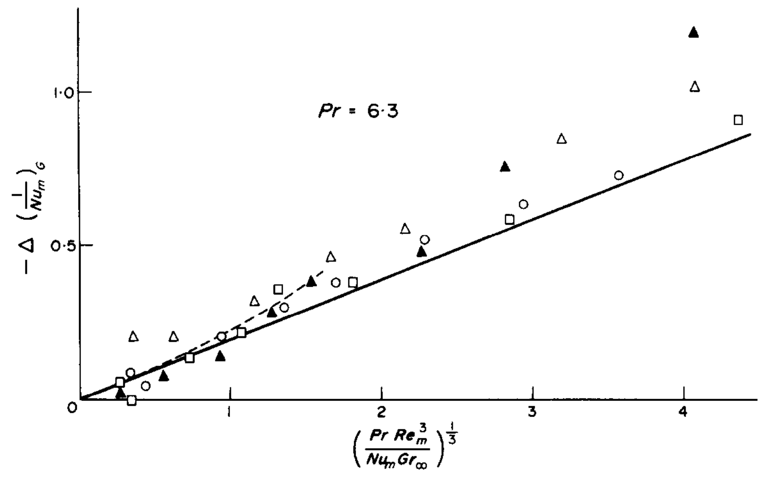
$$\frac{1}{Nu} = 1 - \ln(PrRe) - P_p\frac{NuGr}{PrRe^3}, \quad (5.2)$$

$$PrRe \leqslant 10^{-1}, \quad \frac{NuGr}{PrRe^3} \ll 1. \quad (4.30)$$

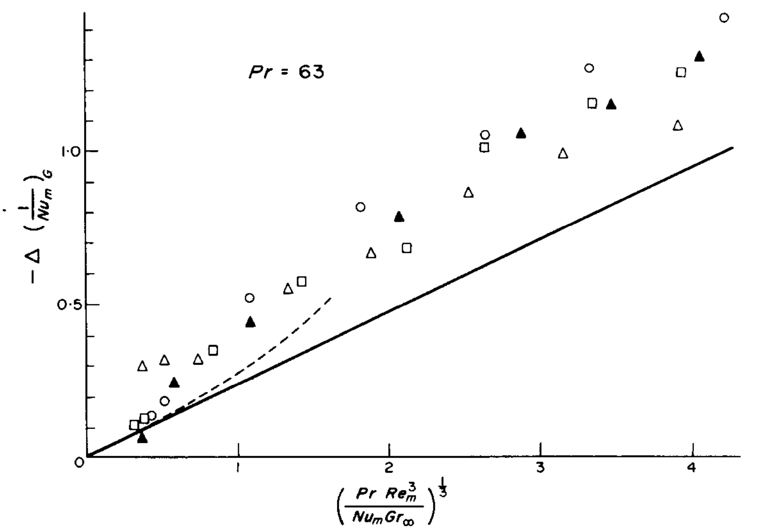
(iii) Experimental results and comparison

The wire was moved upward vertically in the box. In this case, stable voltage drops were not always observed as seen in Fig. 2 (b), although they were relatively stable at smaller Reynolds numbers. This may be accounted for by the fact that the moving wire would overtake the unstationary or turbulent upward wake flow of itself.

The curves of $1/Nu_m$ against $\log(Re_m)$ at a fixed Grashof number are shown in Fig. 8. Asterisks in the figure at $Gr_\infty = 1{\cdot}42 \times 10^{-7}$ and $7{\cdot}5 \times 10^{-6}$ represent the experimental data for wires of the small aspect



(a)



(b)

FIG. 7. Comparison with the experiments by Gebhart and Pera for $Pr = 6{\cdot}3$ and 63 in comparably mixed convection of parallel flow. △, $Pr^2Gr_\infty = 0{\cdot}8 \times 10^{-3}$; $l/a = 16000$; □, $Pr^2Gr_\infty = 0{\cdot}8 \times 10^{-3}$; $l/a = 24000$; ○, $Pr^2Gr_\infty = 0{\cdot}4 \times 10^{-3}$; $l/a = 24000$; ▲, $Pr^2Gr_\infty = 0{\cdot}4 \times 10^{-3}$; $l/a = 32000$.

ratio of $l/a = 5800$ and 5100, respectively. These give appreciably lower values of the inverse Nusselt number. Even with the largest aspect ratio, the end effect was not found to be sufficiently diminished. These results give the difference of the Nusselt numbers, $[\{(Nu_m)_G - Nu_m\}/(Nu_m)_G]_{\max}$, to be about 0·02, which is very small compared with the value of about 0·2 obtained by Hatton, James and Swire [9] at medium Grashof numbers $Gr_\infty = 10^{-3}$ to 10.

The comparison of the analysis and the experiment for free convection with slight contrary forced convection is shown in Fig. 9 for $20000 \leqslant l/a \leqslant 25000$. The theoretical result, the third term of equation (5.1), predicts the difference of the inverse Nusselt number to be about twice the experimental value. This discrepancy may be caused mainly by the deficiency of the aspect ratio. If we make the coefficient Q_p half in equation (5.1),

$$Q_{p-c,\mathrm{ex}} = 0{\cdot}5 \times Q_p, \qquad (5.3)$$

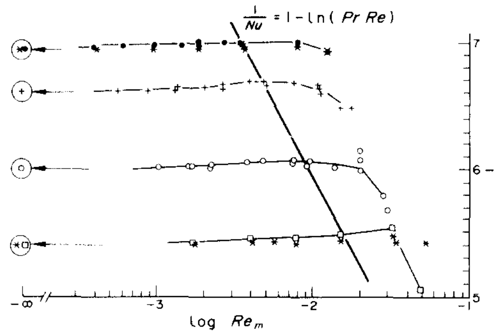


FIG. 8. Heat-transfer correlation in contrary flow. ●, +, ○, □, see the legend of Fig. 4.

the resulted relation can approximately express the experimental results for $20000 \leqslant l/a \leqslant 25000$ in the region of $(PrRe_m^3/Nu_m Gr_\infty)^{1/3} \leqslant 0{\cdot}6$, being shown by the dotted line in Fig. 9.

pure forced convection. Equations and boundary conditions for the zeroth-order approximation are then

$$f_{0c} f_{0c}'' + 2f_{0c}''' = -\xi g_{0c}, \tag{6.4}$$

$$g_{0c} = G_{00} \exp\left(\int_0^\xi -\frac{1}{2} Pr f_{0c}\, d\xi\right), \tag{6.5}$$

$$\left.\begin{aligned} &\xi \to \pm\infty : f_{0c}' \to 1, \quad g_{0c} \to 0; \\ &\int_{-\infty}^{\infty} f_{0c}' g_{0c}\, d\xi = 2\pi \frac{NuGr}{PrRe^3}; \\ &\int_{-\infty}^{\infty} \{f_{0c}'(1-f_{0c}') - h_{0c}\}\, d\xi = 0, \end{aligned}\right\} \tag{6.6}$$

where $F_{0c} = Re^{-1/2}$, $G_{0c} = Gr^{-1}Re^{5/2}$, $H_{0c} = Re^{1/2}$ and $G_{00} \equiv g_{0c}(0)$. The last integral of the flow-drag condition yields

$$h_{0c}(\xi \to \infty) = -h_{0c}(\xi \to -\infty),$$

which is the boundary condition for h_{0c}, signifying an excess pressure difference between the upper and lower

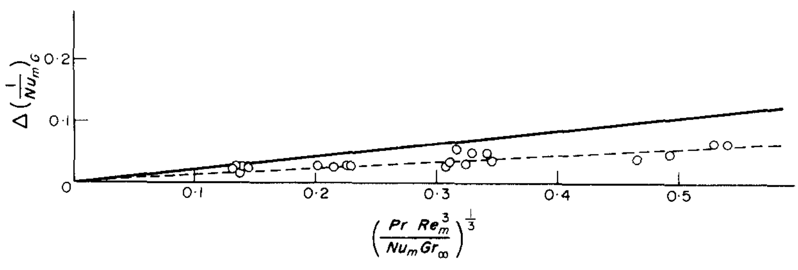


FIG. 9. Comparison of the analysis and the experiment ($20000 \leqslant l/a \leqslant 25000$) in free convection with slight contrary forced convection.

6. CROSS FLOW

(i) *Forced convection with slight cross free convection*

The momentum equation in the main flow direction, namely in the x-axis, has no gravitational term and then dependent variables Φ, t and p in the far field are to be expanded so that the slight bouyancy force in the momentum equation in the y-direction could affect the base flow through the excess pressure p.

$$\Phi = x^{1/2}\{F_{0c} \cdot f_{0c}(\xi) + x^{-1/2} F_{1c} \cdot f_{1c}(\xi) + (x^{-1/2})^2 F_{2c} \cdot f_{2c}(\xi) + \dots\}, \tag{6.1}$$

$$t = x^{-1/2}\{G_{0c} \cdot g_{0c}(\xi) + x^{-1/2} G_{1c} \cdot g_{1c}(\xi) + (x^{-1/2})^2 G_{2c} \cdot g_{2c}(\xi) + \dots\}, \tag{6.2}$$

$$p = \{H_{0c} \cdot h_{0c}(\xi) + x^{-1/2} H_{1c} \cdot h_{1c}(\xi) + (x^{-1/2})^2 H_{2c} \cdot h_{2c}(\xi) + \dots\}, \tag{6.3}$$

where $\xi = Re^{1/2} x^{-1/2} y$. Only the expansion for the excess pressure p is different from that in the case of infinities. When $G_{00} \ll 1$, these equations can be solved by the following re-expansion method with respect to G_{00},

$$f_{0c} = \phi_0(\xi) + G_{00}\phi_1(\xi) + G_{00}^2 \phi_2(\xi) + \dots, \tag{6.7}$$

$$g_{0c} = G_{00}\chi_0(\xi) + G_{00}^2 \chi_1(\xi) + G_{00}^3 \chi_2(\xi) + \cdots \tag{6.8}$$

The zeroth-order approximation for the re-expansions gives the same solution as equations (4.18) and (4.19) for pure forced convection.

$$\phi_0 = \xi, \tag{6.9}$$

$$\chi_0 = \exp\left(-\frac{1}{4} Pr \xi^2\right). \tag{6.10}$$

Equations for the first-order approximation are

$$\xi \phi_1'' + 2\phi_1''' = -\xi \exp\left(-\frac{1}{4} Pr \xi^2\right), \tag{6.11}$$

$$\chi_1 = \exp\left\{\left(-\frac{1}{4} Pr \xi^2\right)\left(\frac{1}{2} Pr \int_0^\infty \phi_1\, d\xi\right)\right\}. \tag{6.12}$$

Here, we assume that the flow has no uniformly upward component, $v(y \to \infty) = -v(y \to -\infty)$, namely $\phi_n(\infty) = -\phi_n(-\infty)$, i.e. that the direction of the whole stream lines of the predominant uniform flow in the horizontal x-axis remains unchanged even if the local flow directions are influenced. The boundary conditions for the first approximation are then given by

$$\left.\begin{array}{l}\xi \to \pm\infty : \phi_1' \to 0, \quad \phi_1'' \to 0; \\ \phi_1(\infty) = -\phi_1(-\infty).\end{array}\right\} \qquad (6.13)$$

Then, equation (6.11) can be solved as follows:

$$\phi_1'' = \frac{1}{\sqrt{Pr(1-Pr)}}\exp\left(-\frac{1}{4}\xi^2\right) + \frac{1}{Pr-1}\exp\left(-\frac{1}{4}Pr\xi^2\right), \quad Pr \neq 1;$$

$$\phi_1'' = \left(\frac{1}{2}+\frac{1}{4}\xi^2\right)\exp\left(-\frac{1}{4}\xi^2\right), \quad Pr = 1. \qquad (6.14)$$

These solutions (6.13) and (6.14) give ϕ_1'' and χ_1 of odd functions and ϕ_1 of an even function with respect to y, which result in zero-average temperature and zero vertical velocity component at $y \to \pm\infty$. Equations and boundary conditions for the second approximation are

$$\xi\phi_2''+2\phi_2''' = \xi\exp\left(-\frac{1}{4}Pr\int_0^\xi \phi_1\,\mathrm{d}\xi\right) - \phi_1\phi_1'', \qquad (6.15)$$

$$\chi_2 = \exp\left(-\frac{1}{4}Pr\xi^2\right)\left\{\frac{1}{8}Pr^2\left(\int_0^\xi \phi_1\,\mathrm{d}\xi\right)^2 - \frac{1}{2}Pr\int_0^\xi \phi_2\,\mathrm{d}\xi\right\}, \qquad (6.16)$$

$$\left.\begin{array}{l}\xi = 0 : \phi_2' = \phi_2'' = 0; \\ \phi_2(\infty) = -\phi_2(-\infty).\end{array}\right\} \qquad (6.17)$$

Integrating equation (6.15) yields

$$\phi_2'' = \exp\left(-\frac{1}{4}\xi^2\right)\left[\int_0^\xi \exp\left(\frac{1}{4}\xi^2\right) \times \frac{1}{2}\left\{\left(\frac{1}{2}Pr\xi\int_0^\xi \phi_1\,\mathrm{d}\xi\right)\exp\left(-\frac{1}{4}Pr\xi^2\right) - \phi_1\phi_1''\right\}\mathrm{d}\xi\right], \qquad (6.18)$$

which gives ϕ_2 of an odd function with respect to y and a secondary suction flow at upper and lower infinities. The value of G_{00} can be determined from the boundary condition (6.6) as

$$G_{00} = \frac{1}{\alpha_1}\frac{NuGr}{PrRe^3} - \frac{\alpha_3}{\alpha_1^4}\left(\frac{NuGr}{PrRe^3}\right)^3,$$

$$\alpha_1 \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty}\exp\left(-\frac{1}{4}Pr\xi^2\right)\mathrm{d}\xi = \frac{1}{2\pi}\sqrt{\left(\frac{\pi}{Pr}\right)},$$

$$\alpha_3 \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty}\exp\left(-\frac{1}{4}Pr\xi^2\right) \times \left\{\frac{1}{8}Pr^2\left(\int_0^\xi \phi_1\,\mathrm{d}\xi\right)^2 - \frac{1}{2}Pr\phi_1'\int_0^\xi \phi_1\,\mathrm{d}\xi + \phi_2' - \frac{1}{2}Pr\int_0^\xi \phi_2\,\mathrm{d}\xi\right\}\mathrm{d}\xi. \qquad (6.19)$$

The stream line of $\Phi = 0$ can be approximately expressed by

$$y = Re^{-1/2}\xi_{\phi=0}\cdot x^{1/2},$$

$$\xi_{\phi=0} = -\left[\phi_1(0)G_{00} + \left\{\frac{1}{2}\phi_1''(0)-\phi_1(0)\phi_2'(0)\right\}G_{00}^3\right]. \qquad (6.20)$$

The circumferential average temperature is thus given by

$$\hat{t} \equiv \frac{1}{2\pi r}\int_{-\infty}^{\infty} t\,\mathrm{d}y = \frac{Re^2}{Gr}(\beta_1\cdot G_{00}+\beta_3\cdot G_{00}^3)r^{-1},$$

$$\beta_1 \equiv \alpha_1,$$

$$\beta_3 \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty}\exp\left(-\frac{1}{4}Pr\xi^2\right) \times \left\{\frac{1}{8}Pr^2\left(\int_0^\xi \phi_1\,\mathrm{d}\xi\right)^2 - \frac{1}{2}Pr\int_0^\xi \phi_2\,\mathrm{d}\xi\right\}\mathrm{d}\xi. \qquad (6.21)$$

Similarly, equations (2.2) and (6.21) are to be joined up smoothly at $r = r_j$.

$$\bar{t} = \hat{t}, \quad \frac{\mathrm{d}\bar{t}}{\mathrm{d}r} = \frac{\mathrm{d}\hat{t}}{\mathrm{d}r}.$$

By use of the approximations $r \ll 1$ and $G_{00} \ll 1$, the unknowns are determined as follows

$$r_j = \frac{1}{PrRe}\left\{1+P_c\left(\frac{NuGr}{PrRe^3}\right)^2\right\}, \quad P_c \equiv \frac{\beta_3-\alpha_3}{\alpha_1^3}, \qquad (6.22)$$

$$\frac{1}{Nu} = 1 - \ln(PrRe)+P_c\left(\frac{NuGr}{PrRe^3}\right)^2. \qquad (6.23)$$

The effect of the slight free convection is also expressed by the parameter $NuGr/PrRe^3$, being proportional to Gr^2. The heat-transfer rate increases slightly with Gr because of $P_c < 0$ as shown in Fig. 10. The restriction for equation (6.22) is similarly written by

$$PrRe \leqslant 10^{-1}, \quad \left(\frac{NuGr}{PrRe^3}\right)^2 \ll 1. \qquad (6.24)$$

In this case, even if the joining point is replaced on the stream line of $\Phi = 0$ instead of the x-axis, the same relation of heat transfer is obtained since equation (6.21) is independent of x.

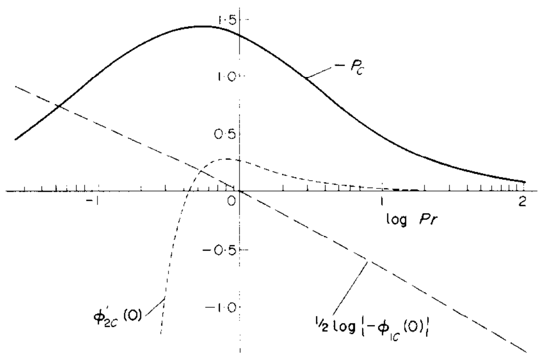


FIG. 10. Calculated results in forced convection with slight cross free convection.

(ii) *Free convection with slight cross forced convection*

For a slight uniform flow in the x-direction added to the predominant free convection in the y-direction, the first-order approximation of the expanded solution in the far field represents the effect of slight cross uniform flow. Therefore, under the restriction of $Rey^{2/5} \ll 1$, the dependent variables are expanded as follows

$$\Psi = y^{3/5}\{\pi_0 \cdot \varphi_0(\eta) + Rey^{2/5}\pi_{1c} \cdot \varphi_{1c}(\eta) + (Rey^{2/5})^2\pi_{2c} \cdot \varphi_{2c}(\eta) + \ldots\}, \quad (6.25)$$

$$t = y^{-3/5}\{\Theta_0 \cdot \theta_0(\eta) + Rey^{2/5}\Theta_{1c} \cdot \theta_{1c}(\eta) + (Rey^{2/5})^2\Theta_{2c} \cdot \theta_{2c}(\eta) + \ldots\}, \quad (6.26)$$

$$p = y^{-4/5}\{\Gamma_{0c} \cdot \gamma_{0c}(\eta) + Rey^{2/5}\Gamma_{1c} \cdot \gamma_{1c}(\eta) + (Rey^{2/5})^2\Gamma_{2c} \cdot \gamma_{2c}(\eta) + \ldots\}, \quad (6.27)$$

where $\eta = (Gr\Theta_0)^{1/4}xy^{-2/5}$. These expansions correspond to the re-expansions of the zeroth-order approximation for the case of pure free convection. From the standpoint of the flow condition on the y-axis, it may be considered that for the vertical velocity and the temperature the zeroth-order approximations should be even functions of x, the first-order be odd and the second-order be even. The zeroth-order approximation gives the same as equations (4.4), (4.5) and (4.6).

The equations and the boundary conditions for the first-order become

$$-\frac{3}{5}\varphi_0\varphi''_{1c} + \frac{4}{5}\varphi'_0\varphi'_{1c} - \varphi''_0\varphi_{1c} = \varphi'''_{1c} + \theta_{1c}, \quad (6.28)$$

$$\frac{3}{5}\varphi_0\theta'_{1c} + \frac{1}{5}\varphi'_0\theta_{1c} + \varphi_{1c}\theta'_0 + \frac{3}{5}\varphi'_{1c}\theta_0 = -\frac{1}{Pr}\theta''_{1c}, \quad (6.29)$$

$$\left.\begin{aligned}&\eta = 0 : \varphi'_{1c} = \theta_{1c} = 0;\\ &\eta \to \pm\infty : \varphi_{1c} \to -1, \quad \varphi_{1c} \to 0, \quad \theta_{1c} \to 0,\end{aligned}\right\} \quad (6.30)$$

where $\pi_{1c} = 1$ and $\Theta_{1c} = Gr^{-1/4}\Theta_0^{3/4}$.

The equations and the boundary conditions for the second-order are

$$-\frac{3}{5}\varphi_0\varphi''_{2c} + \frac{6}{5}\varphi'_0\varphi'_{2c} - \frac{7}{5}\varphi''_0\varphi_{2c} - \varphi_{1c}\varphi''_{1c} + \frac{3}{5}(\varphi'_{1c})^2 = \varphi'''_{2c} + \theta_{2c}, \quad (6.31)$$

$$\frac{3}{5}\varphi_0\theta'_{2c} - \frac{1}{5}\varphi'_0\theta_{2c} + \varphi_{1c}\theta'_{1c} + \frac{1}{5}\varphi'_{1c}\theta_{1c} + \frac{7}{5}\varphi_{2c}\theta'_0 + \frac{3}{5}\varphi'_{2c}\theta_0 = -\frac{1}{Pr}\theta''_{2c}, \quad (6.32)$$

$$\left.\begin{aligned}&\eta = 0 : \varphi_{2c} = \varphi''_{2c} = \theta'_{2c} = 0;\\ &\eta \to \pm\infty : \varphi'_{2c} \to 0, \quad \theta_{2c} \to 0,\end{aligned}\right\} \quad (6.33)$$

where $\pi_{2c} = (Gr\Theta_0)^{-1/4}$ and $\theta_{2c} = Gr^{-1/2}\Theta_0^{-1/2}$.

The stream line of $\Psi = 0$ can be approximately expressed as

$$x = \eta_{\Psi=0} \cdot y^{4/5},$$
$$\eta_{\Psi=0} = -\frac{\varphi_{1c}(0)}{\varphi'_0(0)}\left\{\frac{1}{\pi}\int_0^\infty \varphi'_0\theta_0\,\mathrm{d}\eta\right\}^{2/5} \times \left(\frac{NuGr}{Pr}\right)^{-2/5} Re. \quad (6.34)$$

The circumferential average temperature is then given by

$$\hat{t} \equiv \frac{1}{2\pi r}\int_{-\infty}^{\infty} t\,\mathrm{d}x = A_0\frac{1}{Gr}(NuGr)^{3/5} \times \left[1 + \frac{A_{2c}}{A_0}\left\{Rey^{2/5}(NuGr)^{-1/5}\right\}^2\right] r^{-1}y^{-1/5},$$
$$A_{2c} \equiv \frac{1}{\pi}\left(\frac{1}{\pi}\int_0^\infty \varphi'_0\theta_0\,\mathrm{d}\eta\right)^{-1/5} Pr^{-1/5}\int_0^\infty \chi_{2c}\,\mathrm{d}\eta. \quad (6.35)$$

If equations (2.1) and (6.35) are joined smoothly at $r = r_j$, the unknowns could be given by

$$r_j = C_A(NuGr)^{-1/3}\left\{1 + C_{2AC}\left(\frac{PrRe^3}{NuGr}\right)^{2/3}\right\},$$
$$C_{2AC} \equiv \frac{85}{378}\frac{A_{2C}}{A_0}C_A^{4/5}Pr^{-2/3}, \quad (6.36)$$

$$D = C_B\frac{1}{Gr}(NuGr)^2\left\{1 + \frac{169}{51}C_{2AC}\left(\frac{PrRe^3}{NuGr}\right)^{2/3}\right\}, \quad (6.37)$$

$$\frac{1}{Nu} = \frac{1}{3}\ln E - \frac{1}{3}\ln(NuGr) + Q_c\left(\frac{PrRe^3}{NuGr}\right)^{2/3},$$
$$Q_c \equiv \frac{20}{27}\left(\frac{42}{25}\right)^{2/3} PrA_0^{-1/3}A_{2c}. \quad (6.38)$$

The restriction for equation (6.38) is similarly expressed as

$$Pr^2Gr \leqslant 10^{-3}, \quad \left(\frac{PrRe^3}{NuGr}\right)^{2/3} \ll 1. \quad (6.39)$$

It was found from the numerical computation of these equations that the maximum values of each dependent variable became extremely large compared with the magnitude of the value at the boundary $\varphi_{1c}(\eta \to \pm\infty) = -1$ and that the integral of temperature $\int_{-\infty}^{\infty} \theta_{2c} d\eta$ became too small to be prevented from the computation error, although it seems from the existence of the asymptotic solutions at $\eta \to \infty$ that these equations have a possibility of finite solutions. It implies that secondary suction flow for the zeroth-order approximation at $x \to \pm\infty$ might be still larger than the uniform flow given by the boundary condition for the first-order at $r = r_j$ because of $Rey_j^{2/5} \ll 1$. Therefore, from the viewpoint of physical grounds, the similarity solutions are not necessarily pertinent to the combined flow of this type. The usual profile method cannot be also easily employed because of the difficulty to assume the profile equations for the second-order approximation and the plume width which is able to vary appreciably with Pr.

(iii) *Experimental results and comparison*

The wire was moved horizontally in the box, in this case. The voltage drops had small deviations owing to the moving direction, as seen in Fig. 2 (c), and they were averaged arithmetically to obtain the experimental results. First, we investigated the temperature field of the plume affected by the uniform flow with a thermo-couple of 0·3 mm dia C–Cu. The positions of the maximum temperature of the plume are shown in Fig. 11. It is found that the trace of the maximum

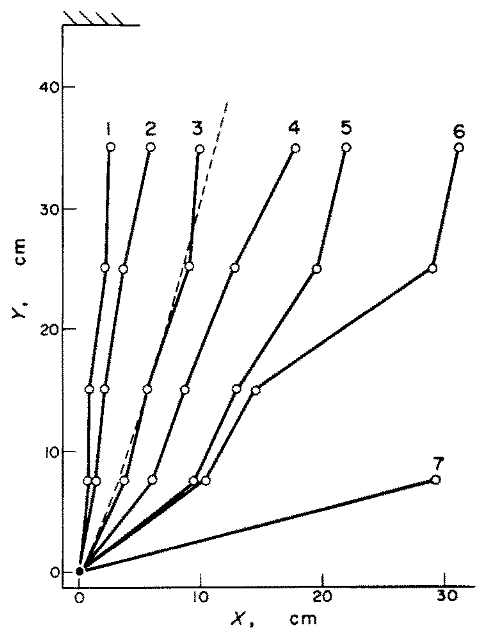


FIG. 11. Trace of the maximum temperatures in the plume at $Gr_\infty \sim 8 \times 10^{-6}$ for $l/a = 22\,000$. 1, $PrRe_m^3/Nu_m Gr_\infty \sim 0{\cdot}0025$; 2, $\sim 0{\cdot}006$; 3, $\sim 0{\cdot}03$; 4, $\sim 0{\cdot}075$; 5, $\sim 0{\cdot}2$; 6, $\sim 0{\cdot}55$; 7, ~ 4.

temperature in the plume may be approximately expressed by $X = K \cdot Y^{4/5}$ of equation (6.34) which is given by the dotted line in Fig. 11, although only the experiment at the smallest Reynolds number can satisfy the restriction (6.39) and $Rey^{2/5} \ll 1$. The curves of $1/Nu_m$ against $\log(Re_m)$ at a fixed Grashof number are shown in Fig. 12 for $20\,000 \leqslant l/a \leqslant 25\,000$. The

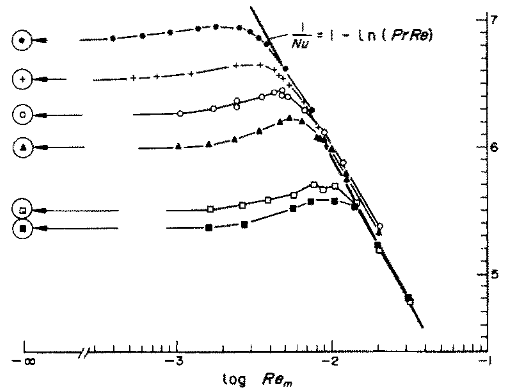


FIG. 12. Heat-transfer correlation in cross flow for $20\,000 \leqslant l/a \leqslant 25\,000$. ●, $a = 0{\cdot}00032$ cm; $Gr_\infty = 1{\cdot}30 \times 10^{-7}$; +, $a = 0{\cdot}00044$ cm; $Gr_\infty = 3{\cdot}2 \times 10^{-7}$; ○, $a = 0{\cdot}00082$ cm; $Gr_\infty = 9{\cdot}1 \times 10^{-7}$; ▲, $a = 0{\cdot}00082$ cm; $Gr_\infty = 1{\cdot}51 \times 10^{-6}$; □, $a = 0{\cdot}00131$ cm; $Gr_\infty = 4{\cdot}8 \times 10^{-6}$; ■, $a = 0{\cdot}00131$ cm; $Gr_\infty = 8{\cdot}0 \times 10^{-6}$.

figure shows the minimum point of the Nusselt number as pointed out by Ower [13].

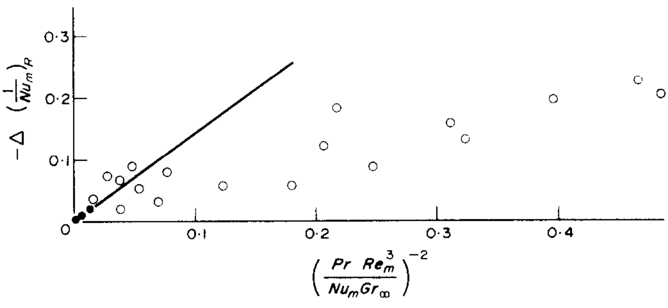
The comparison of the experimental result with equation (6.23) for forced convection affected by slight cross free convection is shown in Fig. 13 (a) for $20\,000 \leqslant l/a \leqslant 25\,000$, where the solid points are the reference points to determine C and the solid line means equation (6.23). The ordinate $\Delta(1/Nu_m)_R$ is given in the similar manner to equation (4.32) by equation (6.23). The experimental points may be regarded to give a rather satisfying relation of equation (6.23) in the region of $(PrRe_m^3/Nu_m Gr_\infty)^{-2} \leqslant 0{\cdot}05$.

For free convection with slight cross forced convection, the experimental points are shown in Fig. 13 (b). If the undetermined coefficient Q_c in equation (6.38) is evaluated at

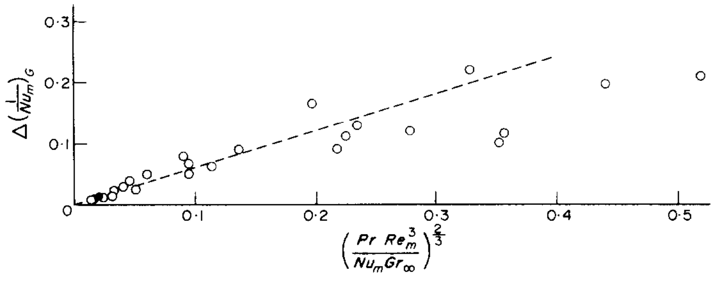
$$Q_{c-\text{ex}} = 0{\cdot}6, \tag{6.40}$$

the experimental results may be roughly expressed by equation (6.38) in the region of $(PrRe_m^3/Nu_m Gr_\infty)^{2/3} \leqslant 0{\cdot}2$ as shown by the dotted line in Fig. 13 (b).

Next, we show in Fig. 14 the experimental results for comparably mixed free and forced convection. The heat-transfer rates were considerably influenced by the aspect ratio l/a as shown in Fig. 15. Even with the largest aspect ratio at the largest Grashof number, the end effect, namely the three-dimensional effect, was not



(a)



(b)

FIG. 13. Comparison of the analyses and the experiments ($20\,000 \leqslant l/a \leqslant 25\,000$). (a) Free convection with slight cross forced convection. (b) Forced convection with slight cross free convection.

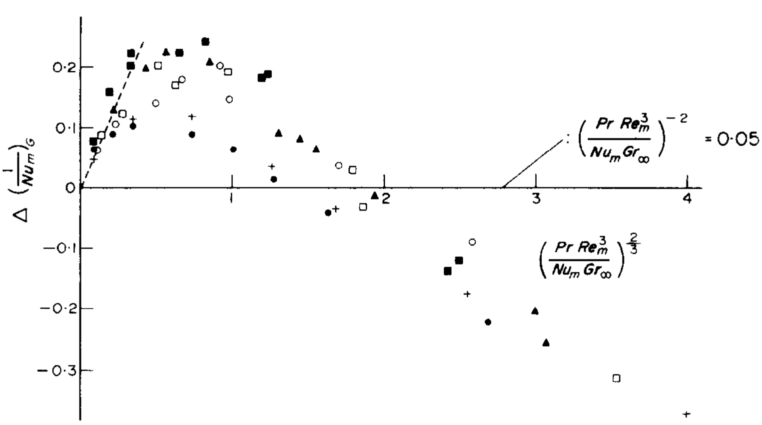


FIG. 14. Comparably mixed convection in cross flow. ●, +, ○, ▲, □, ■, see the legend of Fig. 12.

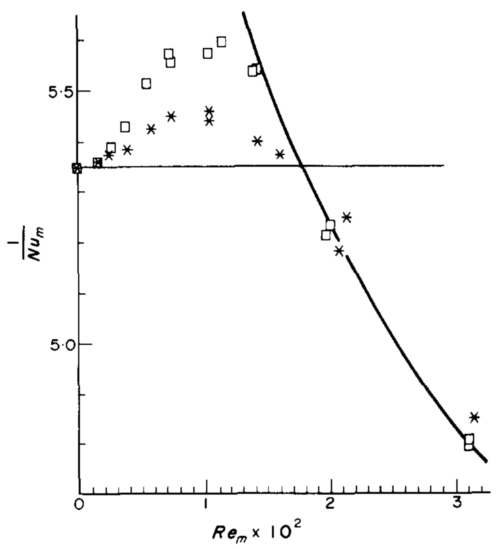


FIG. 15. Example of the influence of the aspect ratio of wire in cross flow at $Gr_\infty = 8{\cdot}0 \times 10^{-6}$. □, $l/a = 22\,000$; $*$, $l/a = 5700$.

sufficiently diminished. The maximum differences of the inverse Nusselt number, $\{\Delta(1/Nu_m)_G\}_{max}$, which are given at the relation of $(PrRe_m^3/Nu_m Gr_\infty) \sim 0{\cdot}6$, increase with increasing l/a and Gr_∞, and then they could be roughly correlated with the product $Gr_\infty^{1/3} \cdot (l/a)$ in the present experiment.

7. CONCLUSION

Heat transfer by mixed free and forced convection with either one of them dominant over the other is analytically investigated for parallel, contrary and cross flows with the expansion and the joining method. The correlation of the heat-transfer rate in the form of $1/Nu_m$ can be systematically expressed as the power function of the parameter of $PrRe^3/NuGr$. These theoretical results were in good agreement with the experimental results made by wires of $l/a = 20\,000$ to $25\,000$ moving in air enclosed in a box. Even in such cases that the agreement is unsatisfactory, the experimental result can be correlated by the same analytical equation with the modified numerical coefficient except for the case of forced convection with slight contrary free convection. For the case of comparably mixed convection in parallel flow, the heat-transfer rate is also expressed pertinently by the parameter of $PrRe^3/NuGr$. For the cases of contrary and cross flows, the heat transfer rates increase with the aspect ratio of the wire at least in the range of 5000 to 40 000.

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